**Aquarius L2 to L3 Processing Document**

Liang Hong, Normal Kuring, Joel Gales and Fred Patt

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1 Introduction

Aquarius Level 3 data are derived geophysical variables that have been aggregated/projected onto an equal area spatial grid over a well-defined time period. From Level 2 data, there are binned, standard map and smoothed map Level 3 data generated in sequence.

Binned

Each Aquarius Level 3 standard binned data product consists of the accumulated data for all L2 products in a product suite, corresponding to a period of time (e.g. daily, 7 days, monthly, etc.) and stored in a global, nearly equal-area, integerized sinusoidal grid.

Standard Map

The Level 3 Standard Mapped Image (SMI) products are created from the corresponding Level 3 binned products. Each SMI file contains a Plate Carrée, pixel-registered grid of floating-point values for a single geophysical parameter ("l3m_data", except for SSS products which contain SSS and two uncertainty values). A color look-up table is also provided in each file that may be used to generate an image from the data.

Smoothed Map

The Level 3 smoothed map products are in the similar format as standard map products except that there is a smoothing process for each pixel (bin) of the map.

2 Binning from Level 2 data

In the binning process, observational values from Level 2 granules that fall within specified time period are accumulated in the closest equal-area bins as determined by the observational center longitude and latitude and then divided by the number of observations in the bins. Simply denoted by,

\[ S_{bin} = \frac{1}{N} \sum_{j=1}^{N} S_j \]  \hspace{1cm} (1)

where \( S_i \) is one observation that falls in the specific qualifiers, i.e. spatial, temporal and quality requirements. The following quality filters are used to mask data in the binning process, POINTING, NAV, LANDRED, ICERED, REFL_1STOKESMOONRED, REFL_1STOKESGAL, TFTADIFFRED, RFI_REGION, SAOVERFLOW, COLDWATERRED,
WINDRED, TBCONS. Detailed information about these masks is found in “Table 1”, the radiometer flags table in L2 format document. [Ref. 1]

The Level 3 binning scheme is based on the sinusoidal map projection. Global distribution of these bins is demonstrated in Fig. 1.

![Fig 1. Demonstration of sinusoidal map with 10deg bins. Each numbered area covers 10deg by 10 deg. [Ref. 2]](image)

Aquarius standard map uses one-degree bins (111 km). So, there are 180 rows and 41,252 bins in total. The daily maps are determined from the L2 granules directly, which then serve as the input for the longer-term maps. So for example, a month’s worth of daily files go into a monthly map which then form the input for a yearly map. This applies to all Aquarius measurement and ancillary Level 3 maps, i.e. salinity, density, spiciness, scatterometer winds and sea surface salinity.

### 3 Smoothing Process

For the Aquarius monthly smoothed maps, all the observations from a month’s worth of L2 granules within 2 degrees of each 1 degree bin (n observations) are used to perform a least-squares fit to a four-parameter bilinear function. The value of this function at the bin-center becomes the smoothed value. Essentially the smoothed map is a spatially averaged standard map. It was chosen as a simplified case of locally polynomial regression among several analyzed smoothing methods [Ref. 3]. This method offers explicit control over smoothness and locality, yet is straightforward to both implement and to interpret.

Eq. 2 defines the bin vector, where lat\(_i\) and lon\(_i\) are its center geolocations. i=0 is the center bin, where the smoothed value is stored. And i=1,...,n are surrounding bins.
\[
bin_{i} = \begin{bmatrix}
X_i \\
Y_i \\
Z_i
\end{bmatrix} = \begin{bmatrix}
\cos(lat_i) \ast \cos(lon_i) \\
\cos(lat_i) \ast \sin(lon_i) \\
\sin(lat_i)
\end{bmatrix}
\]

A rotation matrix is used to transform bin center to north pole for the least distortion in the following interpolation.

\[
rotmat = \begin{bmatrix}
\cos(lon_0) \ast \sin(lat_0) & \sin(lon_0) \ast \sin(lat_0) & -\cos(lat_0) \\
-\sin(lon_0) & \cos(lon_0) & 0 \\
\cos(lon_0) \ast \cos(lat_0) & \sin(lon_0) \ast \cos(lat_0) & \sin(lat_0)
\end{bmatrix}
\]

\[
bin_{i, rot} = rotmat \times bin_i
\]

Then the coordinates are projected to x-y plane to use a bi-linear model for computing the smoothed center bin value. Expressed as,

\[
\hat{S}_0 = (1 \ x_{0, rot} \ y_{0, rot} \ x_{0, rot} \ast y_{0, rot}) \cdot c
\]

where \(S_0\) is the smoothed measurement (e.g. salinity, density and etc), \(c\) is the bi-linear coefficient vector,

\[
c = \begin{bmatrix}
c_0 \\
c_1 \\
c_2 \\
c_3
\end{bmatrix}
\]

Then a least-squares fitting procedure is used to find the coefficients \(c\). The best-fit is found by minimizing the weighted sums of squared residuals, \(\chi^2\),

\[
\chi^2 = (y - Xc)^T W (y - Xc)
\]

where \(y\) is a vector of \(n\) observations, \(X\) is an \(n\) by 4 matrix of predictor variables, and \(c\) is the vector of 4 unknown best-fit parameters, which is to be estimated.

\[
y = \begin{bmatrix}
SSS_1 \\
\vdots \\
SSS_n
\end{bmatrix}
\]

\[
X = \begin{bmatrix}
1 & x_{1, rot} & y_{1, rot} & x_{1, rot} \ast y_{1, rot} \\
\vdots & \vdots & \vdots & \vdots \\
1 & x_{n, rot} & y_{n, rot} & x_{n, rot} \ast y_{n, rot}
\end{bmatrix}
\]

\(W\) is an \(n\) by \(n\) weight matrix, where

\[
W = \begin{bmatrix}
w_1 & \ldots & w_1 \\
\vdots & \ddots & \vdots \\
w_n & \ldots & w_n
\end{bmatrix}
\]
The weight $W$ is calculated in the following steps. First to compute $\theta_j$ (in degrees), the angle between the bin0 and bin$_j$, where $j = 1,..n$. It can be calculated from

$$cs_j = \text{bin}_0 \cdot \text{bin}_j \quad (11)$$

$$\theta_j = \cos^{-1}(cs_j) \quad (12)$$

Then $\theta_j$ is normalized by l2binInput.filter_width,

$$\text{ratio}_j = \theta_j / \text{l2binInput.filter\_width} \quad (13)$$

weight for each bin is then defined as following for bilinear regression,

$$w_j = 1 - \text{ratio}_j \times \text{ratio}_j \quad (14)$$

After the bilinear coefficients vector $c$ is found, the smoothed value of center bin is easily computed from Eq. 5. This procedure applies to all Level 3 values, salinity, density, spiciness, scatterometer winds and sea surface salinity.

While the uncertainty fields ("SSS_ran_unc" and "SSS_sys_unc") in SSS L3 smoothed map products are computed from L2 directly without standard binning process. See section “2.3 Uncertainty Propagation in L3 Averaging” in reference document [Ref. 4] for equations computing these two L3 uncertainty fields.

4 References
1. Aquarius_Level-2_Data_Products_5.0
2. https://oceancolor.gsfc.nasa.gov/docs/format/l3bins/
4. memo_uncertainties_V2